FRIEDRICH-SCHILLER-
UNIVERSITÄT
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Department of Mathematics Dr. Alexander Fromm
Stochastic Analysis (SS 2019)

Exercise sheet 11

Submission: 25.06.2019

Problem 1 - Gronwall's lemma (4 Points)

Let $g : [0, \infty) \to \mathbb{R}$ be a continuous function satisfying

$$0 \leq g(t) \leq a + b \int_0^t g(s)ds, \qquad \text{for all } t \geq 0,$$

where $a, b \geq 0$ are constants. Show that $g(t) \leq ae^{bt}$.

Hint: Consider the function $t \mapsto e^{-bt} \int_0^t g(s)ds$.

Problem 2 (3 Points)

Let $(B_t)_{t \geq 0}$ be a Brownian motion. Show that

$$X_t = e^{-\theta t} \left(x + m \left(e^{\theta t} - 1 \right) + \sigma \int_0^t e^{\theta s} dB_s \right)$$

is a solution to the SDE

$$dX_t = \theta(m - X_t)dt + \sigma dB_t, \quad X_0 = x,$$

where $x, m \in \mathbb{R}$ and σ, $\theta > 0$. Is the solution unique?

Problem 3 **(4 Points)**

Let $(B_t)_{t\geq 0}$ be a Brownian motion. Give three different solutions of the SDE

$$dX_t = \mathbb{1}_{\mathbb{R}\setminus\{0\}}(X_t)\,dB_t, \quad X_0 = 0,$$

and prove your answer.

Total: 11 Points

Terms of submission:

- Solutions can be submitted in groups of at most 2 students.
- Please submit at the beginning of the lecture or until 9:50 a.m. in room 3523, Ernst-Abbe-Platz 2.